S.4 INTERSCHOOLS VIRTUAL MATHEMATICS SEMINAR 2020

DATE: 16th Nov -5th Dec.2020

1. Below are lengths in cm of pencils used by 50 primary two pupils of a certain school.

5.4	4.7	7.0	6.1	3.4	5.2	3.2	4.5	5.9	6.8
6.7	5.8	8.5	6.5	8.2	5.4	4.2	6.5	4.1	7.5
7.1	5.2	5.4	6.9	5.7	8.1	6.4	6.1	7.2	6.2
3.7	5.1	5.6	5.0	7.0	7.9	5.4	7.1	3.5	7.2
8.3	6.5	5.7	6.0	5.1	8.7	5.3	4.6	6.9	6.7

a) Construct a frequency distribution table with equal class width starting 3.0 – 3.9, 4.0 – 4.9 ,----

b) Calculate the mean length using an assumed mean of 6.45.

c) State the (i) class width

(ii) modal class

(iii) median class

d) Draw a Histogram to represent the information and use it to estimate the mode.

e) Construct a cumulative frequency curve and use it to estimate the median'

2. In a certain school there are 50 students who play three games namely, Chess, Tennis and Volleyball, 24 play Chess, 26 play Tennis and 29 play Volley ball, 9 play both Chess and Volleyball, while 13 play both Tennis and Volleyball, 11 play both Chess and Tennis.

Each of these students play at least one of the three games

a) Represent the above information on a Venn diagram

b) Find

i. How many students play all the three games?

ii. The number of students who play only one game

iii. The probability that a student selected at random from class plays only Tennis

iv. The probability that a student selected at random from the class plays at most two of the games

3. Quadrilateral ABCD was mapped onto A^{II} (-4, 6), B^{II} (-4, 10), C^{II} (0, 8) and D^{II} (2, 4) after an enlargement of scale factor 2 about the origin followed by a negative quarter turn about the origin.

(a)Write down the matrix for the:

(i) enlargement.

(ii) rotation described above.

(b) Use the above matrices to find the co-ordinates of points A, B, C, and D.

(c) If the area of the image quadrilateral $A^{II}B^{II}C^{II}D^{II}$ is 36 square units, find the area of the object ABCD.

4. (a) Given that y varies inversely as the square of x and that when x = 4, y = 1, find x when y = ¼.

(b) The time taken to conduct a staff meeting partly varies as the number of members present and partly as the numbers of the items on the agenda. When 20 members are present, with an agenda of 5 items the meeting lasts 1hr 10mins. When 30 members attend with only 4 items on the agenda, the meeting takes 1hr. 38 mins.

a) how long would a meeting with 10 items on the agenda attended by 20 members last?
b) how many items are on the agenda for a meeting that lasts 2hr. 40 mins. when 50 members attend ?
c) how many members were present in a meeting that lasted 1 hr. 20 mins. in a meeting of 7 items on the agenda?

5. (a) A coin and a die are tossed simultaneously once

i) Draw a table to show all the possible outcomes
ii) What is the probability that a head and a triangle number show up?

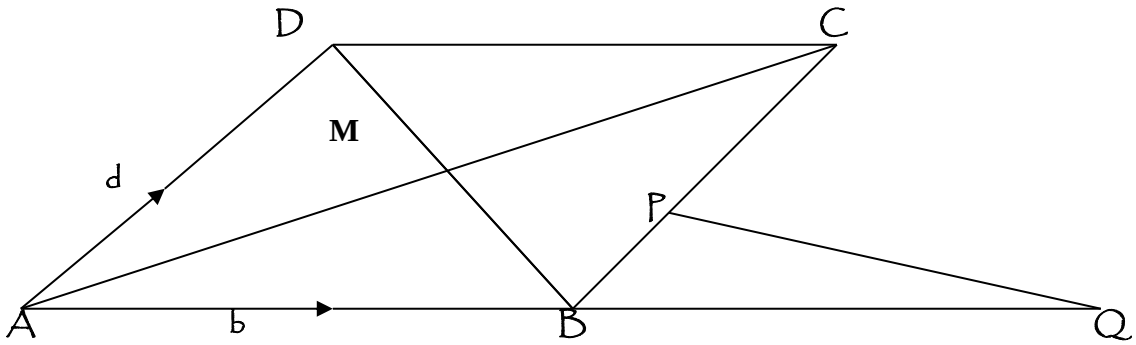
(b) A bag contains 6 balls, 4 of which are red, and the rest are blue. Two balls are selected one at a time. If a red one is selected first, it's put back to the bag, and if it's blue it's not replaced. Draw a probability tree to show the possible outcomes. Find the probability that the second ball picked is:

(i) blue given that the first is red.

(ii) red given that the first was blue.

(iii) blue.

6. The diagram below is a parallelogram. 2**BC** = 3**PC**, **AQ** = 2**AB**, **AB** = **b**, and **AD** = **d**.



(a) Express in terms of **b** and **d** : **AC**, **BD**, **BP**, **AP**, **PQ**

(b) If M is the intersection of AC and DB, show that the three points **M**, **P** and **Q** are collinear.

7. The tax rates in a certain country for government employees are as follows:

Taxable income (shs.)	**Rate (%)**
80,001 – 190,000	5.0
190,001 – 280,000	7.5
280,001 – 380,000	12.0
380,001 – 490,000	15.0
Above 490,000	20.0

Every employee is entitled to the following allowances:

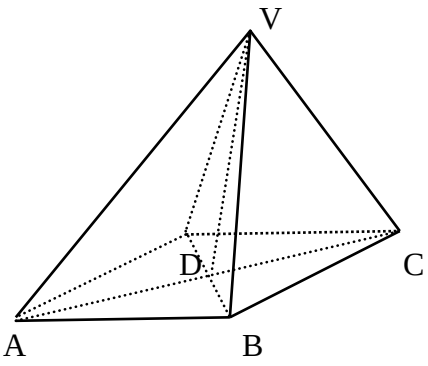
Medical	–	720,000 per annum
Electricity	–	40,000 per month
Transport	–	2,500 per day
Housing	–	90% of his monthly medical income.

If the employee paid a monthly income tax of shs. 125,350 in June 2013, calculate;

(a). the employee's taxable income.

(b). the employee's monthly gross income.

8. The figure below shows a right pyramid VABCD with a square base ABCD whose area is $144cm^2$, the height of the triangular face is 8cm.



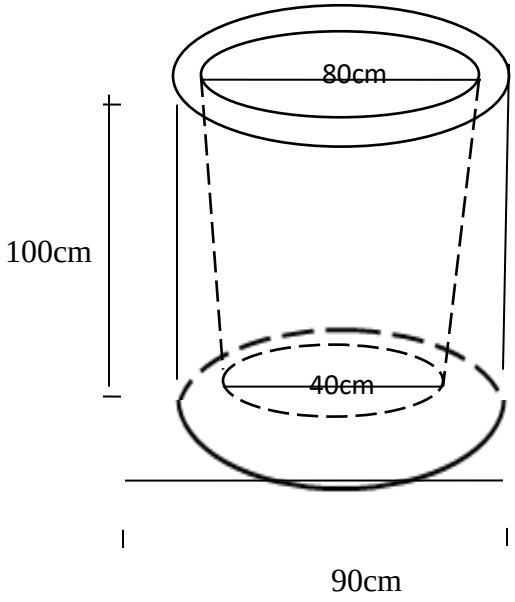
(a) Find the:

(i) volume of the pyramid (ii) surface area of the pyramid

(b) Angle between

(i) plane VAD and base. (ii) plane VAC and line VB.

9. A vase is made with the dimensions shown below



(a) Find the volume of;

(i) water which can fill the vase.

(ii) material which was used to make the vase.

(b) Find the total surface area of the vase.

(c) If a tap is opened to fill the vase at $50cm^3$/minute, how long will it take to fill the vase?

10. a) A motor cyclist travelled 8km uphill at a speed of $x$km/hr. On the return journey down the hill, his speed was $(x + 4)$ km/hr. The difference in time between the uphill and downhill journeys was 10minutes

i. Write down an expression for the time taken for the uphill journey and downhill journey
ii. Using the expressions above, form a quadratic equation for the difference in time for the two journeys
iii. Solve the quadratic equation
iv. What was his average speed for the uphill and downhill journeys?

b) Otim and Mukasa wish to travel to the next trading centre which is 30.8km away. They will travel by their bicycles. When Otim had covered 9km, travelling steadily at $4kmh^{-1}$, Mukasa started riding at a steady speed of $7km^{-1}$ from where Otim started. Both Mukasa and Otim maintained their cycling speeds until Mukasa overtook Otim

i. Represent Otim's and Mukasa's journeys on the same distance-time graph. Hence find the time and distance at which Mukasa over took Otim
ii. Given that Mukasa then reduced his speed and maintained the new speed till he arrived at the trading centre, there by arriving 0.6 hours later than if he had maintained the $7km^{-1}$ speed
iii. Calculate by how much he reduced his speed
iv. For how long was he in the trading centre before Otim joined him

S.4 MATHEMATICS SEMINAR 2020
DATE: 16th Nov -5th Dec.2020
PRACTICE QUESTIONS

PAPER ONE

Equations and Quadratic Graphs

1. a) Solve the equation by factorization: $3x^2 - 10x + 3 = 0$
   b) Form a quadratic equation in x whose roots are -3 and ¼
   c) Express $x^2 + x - 12$ in the form $(x + a)^2 + b$. Hence solve the equation $x^2 + x - 12 = 0$

2. a) Draw a table for $y = (3x + 1)(2x - 5)$ over the interval $-1 \leq x \leq 4$
   b) Use your table to draw a graph of $y = (3x + 1)(2x - 5)$ with a scale of 2cm:1unit horizontally and 2cm:5units vertically
   c) Draw on the same axes the line $y = 5$
   d) Use the two graphs above to solve the equation $6x^2 - 13x - 10 = 0$
   e) State the minimum value of y and the value of x for which it occurs

Matrices

3. a) If M is a 2x5 matrix and N is 5x3 matrix, state the order of the product matrix MN
   b) Given the matrices $P = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 27 & 37 \\ 5 & 5 \end{pmatrix}$, find matrix Q such that PQ=R
   c) Given that matrix $T = \begin{pmatrix} x+1 & x+1 \\ -3 & 2x \end{pmatrix}$ is singular, find the possible values of x
   d) Use matrix method to solve the equations $2y - 4x + 2 = 0$
   $$3x - 2y = 5$$
4. Four students Alex, Ben, Charles and Dan went to a stationary shop. Alex bought 4 pens, 6 counter books and 1 graph book. Ben bought 10 pens, and 5 counter books. Charles bought 3 pens and 3 counter books. Dan bought 5 pens, 2 counter books and 8 graphs books
   The cost of a pen, a counter book and a graph book were shs.400, shs.1200 and shs.1000 respectively

a) i) Write a 4x3 matrix for the items bought by the four students
   ii) Write a 3x1 matrix for the cost prices
b) Use the matrices above to calculate the amount of money spent by each student
c) If each student was to buy 4 pens, 10 counter books and 6 graph books, how much money in total would be spent by all the four students?

Statistics

5. The table below shows the marks obtained by students in a certain school

Class	30-39	40-49	50-59	60-69	70-79	80-89	90-99

frequency	4	6	8	12	10	7	3

(a) Draw a histogram and use it to estimate the mode.

(b) Draw an O-give and use it to estimate

i The median

ii The 80^{th} percentile

iii The pass mark if 15 students passed

6. The table below shows ages of 50 people treated for Tuberculosis (T.B) at a health centre

86	85	56	59	80	62	63	50	91	62
56	27	50	54	67	61	52	52	16	28
66	46	55	58	56	77	26	40	42	51
35	45	68	51	49	40	93	84	79	63
52	53	25	93	27	71	66	52	30	12

a) Starting with 10 as the lowest mark, construct a frequency distribution table using equal class width of 10 marks

b) Use the frequency table to calculate the mean age of the people treated for T.B using a working mean of 54.5 years

c) Calculate the median age of people treated for T.B

Transformation and matrices of transformation

7. A triangle whose vertices are P, Q and R is mapped on to a triangle whose vertices are $P^1(0,1)$, $Q^1(5,7)$ and $R^1(0,2)$ by a matrix of transformation $T_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. The triangle $P^1Q^1R^1$ is then mapped on to triangle $P^{11}Q^{11}R^{11}$ by a matrix of transformation $T_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

a) Describe the transformations T_1 and T_2 above fully.

b) Find the coordinates of P^{11} , Q^{11} and R^{11}

c) Find the single matrix of transformation which would map triangle P^{11} $Q^{11}R^{11}$ back on to triangle PQR.

d) Find the co-ordinates of P, Q and R

e) Use the determinant of T_2T_1 to find the ratio of the area of triangle PQR to the area of triangle $P^{11}Q^{11}R^{11}$

8. A triangle ABC with vertices A(1,3), B(3,3) and C(3,1) is enlarged with a scale factor of -4 about (2,2) to triangle $A^1B^1C^1$.

Triangle $A^1B^1C^1$ is then rotated through a positive quarter turn about (0,-4) to triangle $A^{11}B^{11}C^{11}$.

a) Draw on the same axes the triangles ABC, $A^1B^1C^1$ and $A^{11}B^{11}C^{11}$

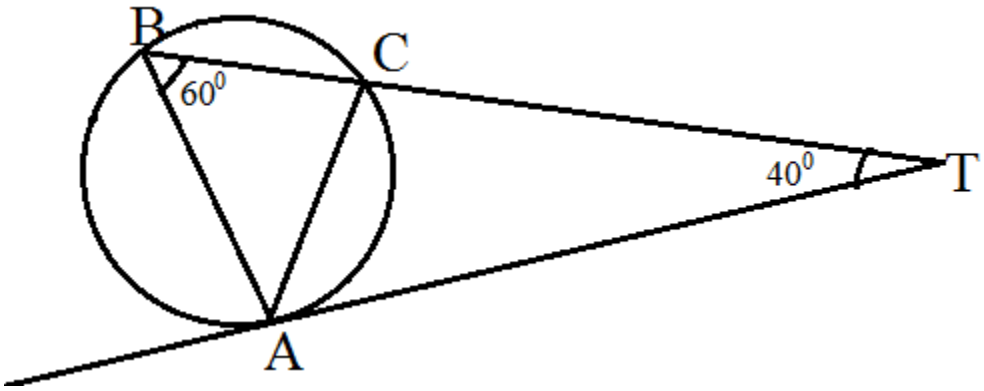
b) Write the co-ordinates of

i. A^1, B^1 and C^1

ii. A^{11}, B^{11} and C^{11}

Circle properties

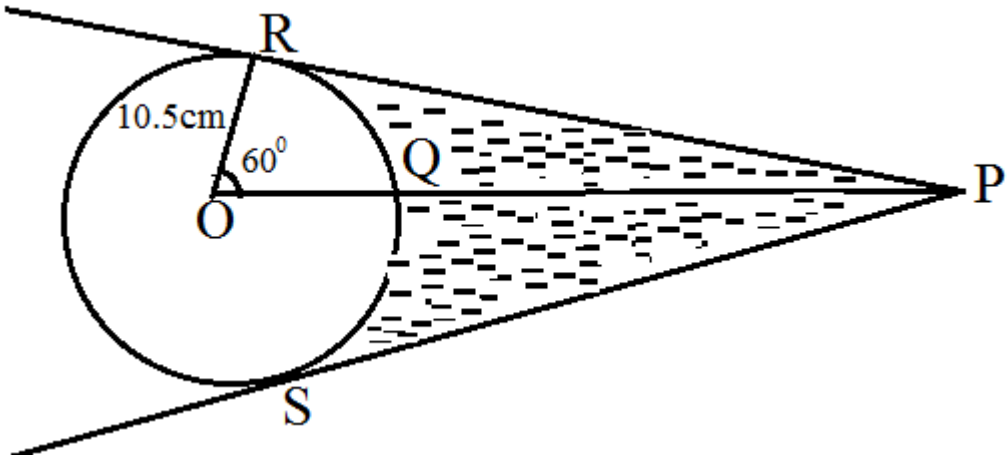
9. In the figure below, $\overline{TA}$ is a tangent to the circle and BCT is a straight line. Angle $ABC = 60^0$ and angle $ATC = 40^0$



a) Find angle
   i. ACT
   ii. BAC

b) Given that $\overline{AT}$=8cm and $\overline{CT} = 6$ cm, calculate lengths;
   i. $\overline{BT}$
   ii. $\overline{BA}$
   iii. $\overline{AC}$

10. The diagram below shows a circle with centre O and radius 10.5cm. Two tangents PR and PS are drawn from a point outside the circle. Angle POR=60^0 and PO intersects the circle at Q



Calculate the

a) Lengths of the tangents

b) Area bounded by the tangents and the arc SQR [use $\pi = \frac{22}{7}$]

Probability

11.(a) A fair die is tossed once and what appears on the top face is recorded. Find the probability that a triangular number shows up.

b) Irama box containing marbles, the probability of picking a red marble is $\frac{2}{5}$ and the probability of picking a blue marble is $\frac{3}{7}$. What is the probability of picking a green marble from the same box.

c) A bag contains 3 black balls, 4 green balls and 5 yellow balls

i. If two balls are picked at random without replacement, find the probability that both are of the same colour

ii. How many black balls must be added to the bag so that the probability of drawing a black ball is ½?

Trigonometry

12.a) Given that $\sin\alpha = -\frac{3}{4}$ and that α is reflex, find the possible values of

i. $\sin\alpha + \tan\alpha$

ii. $2\cos^2\alpha - 3\tan^2\alpha$

b) Find the area of a triangle ABC where a=5cm, b=7cm and c=8cm

c) Without using tables or calculators find the values of

i. $\cos 240^0$

ii. $\tan 315^0$

iii. $\sin 1860^0$

d) Solve the equation $\cos\theta + \sqrt{3}\sin\theta = 1$ for $-180^0 \leq \theta \leq 180^0$

13.a) At a certain point on the level ground, the angle of elevation of the top of a tower T is 28^0. At another point 100metres away from the first point, the angle of elevation is 35^0. Find the two expressions for the height of the tower, hence find the height of the tower and give your answer to the nearest metre

b) A dog tied on a stick rope 4.5m long is tethered to a tree stamp 2.5m from a straight path. For what distance along the path is one in danger of being bitten by the dog?

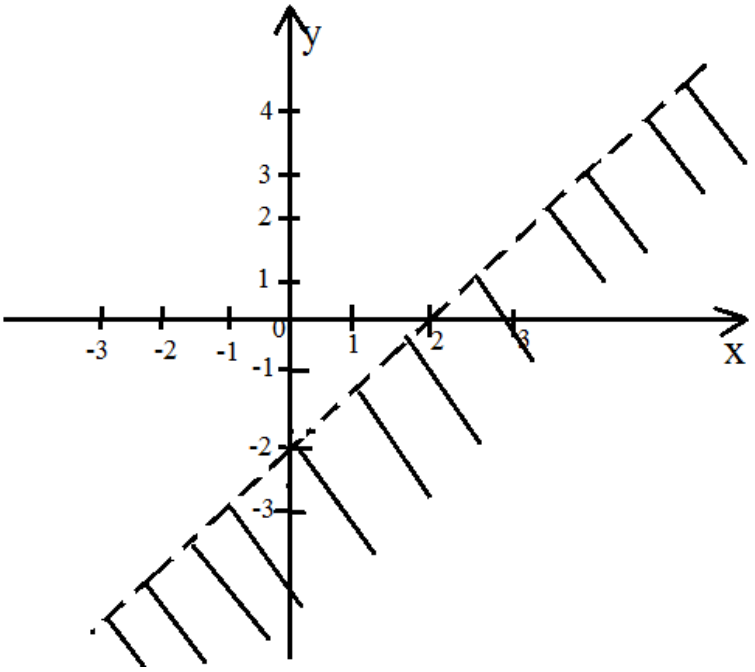
Inequalities and linear programming

14. a) Given that A=$\{x: -2 \leq x \leq 1\}$ and B$\{x: 0 < x \leq 5\}$, represent A ∩ B on a number line

b) Solve for x in the equality $(x+2)(x-4) < x^2 - 6$

c) Show by shading the unwanted region the inequalities $x + 3y > 6, 2x - 3y \leq 9$

d) Determine an inequality which is represented by the un-shaded region on the graph below



15. Peter is contracted to transport 45 tonnes of lake sand to a building site. He has 6 trucks which can each carry 3 tonnes and has 7 tippers which can each carry 1 tonne of lake sand. The tipper makes 5 trips per day and the truck makes 3 trips per day. He has 9 drivers available. The cost of a tipper is shs. 40,000 per day and a truck is shs.100,000 per day. By letting x and y be the number of tippers and trucks used respectively;
    a) Form six inequalities representing the above information
    b) Represent all the above inequalities on the same graph by shading off the unwanted region
    c) Find the number of trippers and trucks he has to use in order to minimize the total cost

Paper Two

PROPORTIONS AND VARIATION

16a) The kinetic energy e, varies jointly as the mass m, and the square of the velocity,v.

A mass of 8kg and a velocity of $5ms^{-1}$ has a kinetic energy of 100J, find the kinetic energy for a mass of 6kg and velocity of $9ms^{-1}$.

b) The daily cost per child in a Kampala family is partly constant and partly inversely proportional to the number of children in the family. Given that the cost per child for a family of 10 children is shs 350 and for a family of 20 children is shs 300,
Find the cost per child for a family of
i. 50 children
ii. n children

RELATIONS, MAPPING AND FUNCTIONS

17a) Given the function $f(x) = ax^2 + bx$ and that $f(1) = -2$ while $f(2) = 2$, find the values of a and b

b) Given the function $g(x) = \frac{2x-3}{x^2-3x-10}$, find the values of x for which $g(x)$ is meaningless

c) Given the functions $f(x) = \frac{x+3}{2}$, $g(x) = \frac{1-3x}{5}$, determinie the value of x for which $fg(x) + gf(x) = 0$

d) Find $g(3)$ given that $g^{-1}(x) = \frac{x+1}{x}$

18a) Given that $x \rightarrow 3x + 5$ represents a function, find the range for the domain

{-2,-1,0,1,2,3}

b) Given that $x \rightarrow 5x + 2$ represents a function, find the domain if the range is

{27, 7,5,0,-20}

c) Given that T={2,5,6,8,9,10,12,13}. Illustrate on papygrams the relations::

i. "greater than by 3"

ii. "factor of"

SETS

19. 100 people seated at a table in a restaurant were asked if they had ordered for any of the following items: Margaritas, Chillcoqueso or Quesadilllas. 23 had ordered for none of the items, 11 had ordered for all the three items, 29 had ordered for Chillconqueso or Quesadillas but did not order for Margaritus, 41 had ordered for Quesadillas, 46 had ordered for at least two of these items, 13 had ordered for Magaritus and Quesadillas but not Chillconquesso, 26 people had ordered for Magaritus and Chillconquesso.

(a) Draw a Venn diagram to represent this information.

(b) How many people had ordered for Chillconquessso?

(c) How many people had ordered for either Chillconquesso or Magaritas but not Quesadillas?

20. In a certain school, a sample of 100 students was picked randomly. In this sample, it was found out that 78 students play Netball (N), 82 play Volleyball (V), 53 play Tennis (T) and 2 do not play any of the three games. All those that play Tennis also play Volleyball. 48 play all the three games

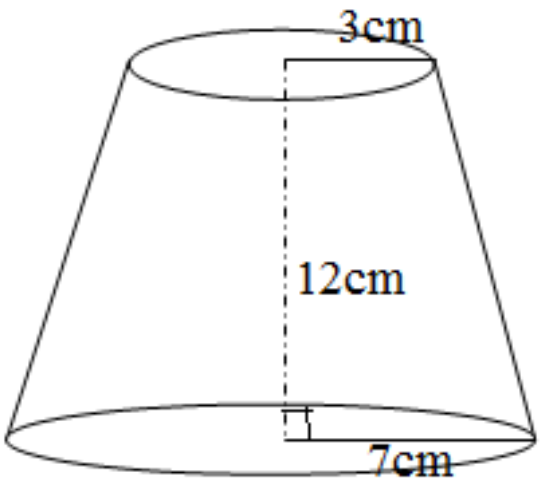
a) Represent the given information on a Venn diagram

b) How many students play both Netball and Volleyball but not Tennis?

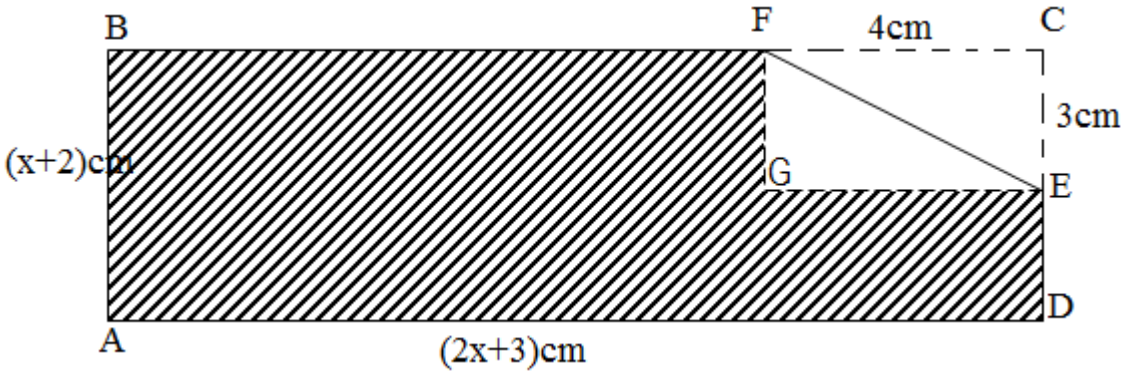
c) If a student is picked at random from the sample, what is the probability that the student plays two games only?

MEASURATION

21. a) The lampshade show below has radii 3cm and 7cm and height 12cm. Find its total surface area if only the lower end is open.



b) The figure below shows are rectangular piece of paper ABCD which has been folded along EF such that C maps onto G



Given that $\overline{EC} = 3cm$ and$\overline{FC} = 4cm$, $\overline{AB} = (x + 2)cm$ and $\overline{AD} = (2x + 3)cm$

Find

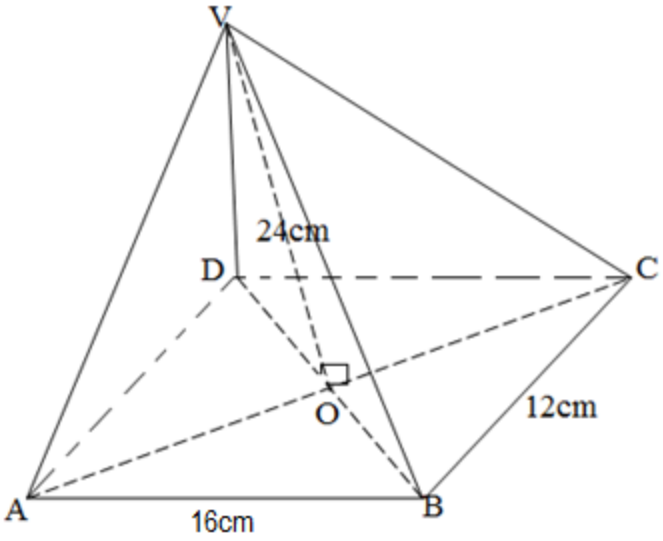
i. The area of triangle ECF

ii. An expression for the area of the shaded region ABFGED in terms of x

iii. If the shaded area is 43cm^2, show that $2x + 7x - 49 = 0$. Hence find the length of $\overline{AD}$

THREE DIMENSIONAL GEOMETRY

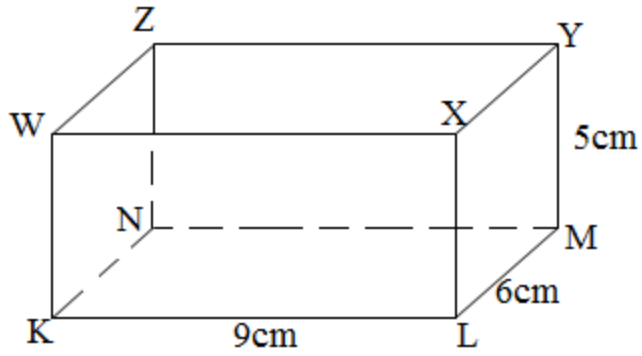
22.a) The figure below shows a right pyramid on a rectangular base ABCD



Calculate the

i. Volume of the pyramid
ii. Angle between $\overline{AV}$ and the base ABCD
iii. Angle between the planes ADV and BCV

b) The diagram below shows a cuboid KLMNWXYZ in which $\overline{KL} = 9cm$, $\overline{LM} = 6cm$ and $\overline{MY} = 5cm$



i. Calculate the length $\overline{KM}$ and $\overline{KY}$
ii. Determine the angle between line $\overline{KY}$ and the base KLMN
iii. Determine the angle between the planes KZYL and WXY

BUSINESS MATHEMATICS

23. The table below shows income tax rates of a certain country for government employees. This is applied after all allowances have been deducted

Monthly taxable income	Taxation rate (%)
First 100,000	5
Next 100,000	10
Next 150,000	20
Next 200,000	25
The rest	30

Alex a civil servant of that country pays 960,000 as yearly income tax (PAYE) and his total monthly allowance is shs. 110,000

Determine his

i. Monthly taxable income
ii. Annual gross income

b) The monthly income tax system of a country is given as below

Basic pay (Ushs)	Tax (%)
150,001-250,000	10.0
250,001-350,000	12.5
350,001-450,000	16.0
450,001-550,000	22.5
550,001-650,000	30.5
Above 650,000	32

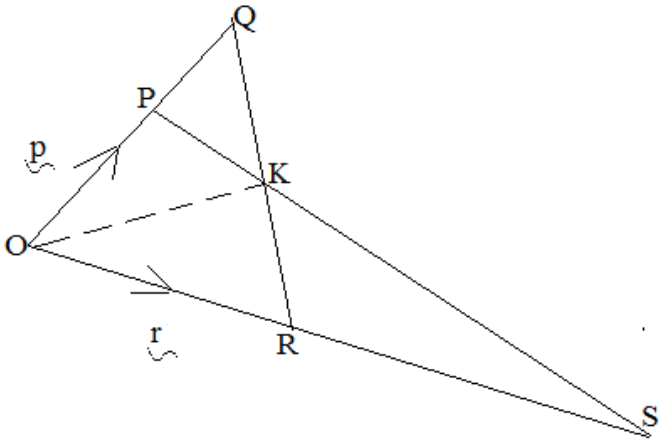
An allowance in excess of shs. 80,000 is subjected to a tax of 25% of the monthly allowance. Two employees A and B are such that A earns a basic monthly pay of shs. 355,000 and a top up allowance of shs.185,000 per month while B earns only a basic monthly pay of shs. 540,000

i. Who of the two employees pay more monthly income tax than the other and by how much?
ii. Express employees A's income tax a percentage of his monthly earnings

VECTORS

24 a) A(3,2) and B(-1,4) are two points on the L. A point P divides AB in the ration 2:1. Determine the coordinates of P

b) In the figure below, vector $\overset{OP}{\sim} = \overset{p}{\sim}$ and $\overset{OR}{\sim} = \overset{r}{\sim}$. Vector $\overset{OS}{\sim} = 2\overset{r}{\sim}$ and $\overset{OQ}{\sim} = 2\overset{p}{\sim}$

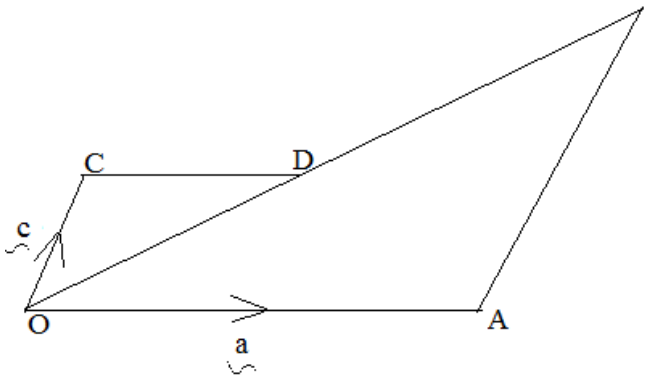


i. Express in terms of $\overset{p}{\sim}$, and $\overset{r}{\sim}$ $\overline{QR}, \overline{PS}$

ii. The line $\overline{QR}$ and $\overline{PS}$ intersect at K such that $\overline{QK} = m\overline{QR}$ and $\overline{PK} = n\overline{PS}$ where m and n are scalars. Find two distinct expressions for $\overset{OK}{\sim}$ in terms of $\overset{p}{\sim}$, $\overset{r}{\sim}$, $\overset{m}{\sim}$ and $\overset{n}{\sim}$ hence find the values of m and n

iii. State the ration $\overline{PK}:\overline{KS}$

c) In the figure below, vectors $\overset{OA}{\sim} = \overset{a}{\sim}$ and $\overset{OC}{\sim} = \overset{c}{\sim}$, $\overline{CD} = \frac{1}{3}\overline{OA}$ and $\overline{AB} = 3\overline{OC}$



i. By expressing vectors in terms of $\overset{a}{\sim}$ and $\overset{c}{\sim}$ find $\overline{OD}, \overline{AB}$ and $\overline{OB}$

ii. Show that points O, D and B are collinear

iii. Using vectors, show that the points P(-4, 1), Q(0,2) and R(8,4) lie on a straight line

End